Abstract

In resent years, dynamical systems has had many applications to science and engineering; these include mechanical vibration, lasers, biological rhythms, super conducting circuits, insect outbreaks, chemical oscillators, genetic control systems, chaotic water wheels, and even a technique for using chaos to send secret messages. Some of which have gone under the related headings of [nonlinear analysis]. Behind these applications there lies a rich mathematical subject; which we will treat one of them in this thesis. [2].

This subject centers on the orbits of iteration of a nonlinear rational difference equation. In particular, we are interested in the analytic analysis (e.g. the local analysis near a fixed point, the character of semicycles and global asymptotic stability theory). Although the subject has analytic analysis, a geometric or topological flavor plays an important role for suggesting the behavior of this rational difference equation.

In this thesis, we will investigate the nonlinear rational difference equation

$$x_{n+1} = \frac{\beta x_n + \gamma x_{n-k}}{B x_n + C x_{n-k}}, \qquad n = 0, 1, \ldots \quad (1)$$

where the parameters β, γ and B, C and the initial conditions $x_{-k}, \ldots, x_{-1}$ and x_0 are nonnegative real numbers, $k = \{1, 2, 3, \ldots\}$.

Our concentration is on invariant intervals, periodic character, the character of semicycles and global asymptotic stability of all positive solutions of equation(1).

In order to investigate the global attractivity, boundedness, periodicity, and global stability of solution of this difference equation, we will use MATLAB to see how the behavior of this difference equation look like. MATLAB now capable of finding approximations of solutions of this difference equation and also producing high quality graphics representations of its behavior. Although MATLAB are a wonderful tool for suggesting the behavior of this difference equation; it is the base on which to build the mathematical theory, it do not normally provide proof of its existence in the strict mathematical sense. We will use different techniques to help us solving this difference equation and prove it.

There have been several papers and monographs on the subject of Dynamical Systems. There are several distinctive aspects which together make this thesis unique.
• First of all, the results of this thesis solve the open problem 6.10.17 (equation(6.100)) proposed by Kulenvic and Ladas in their monograph [Dynamics of Second Order Rational Difference Equations: with Open Problems and Conjectures, Chapman & Hall/CRC, Boca Raton, 2002]. [7]

• Second, this thesis treats the subject from a mathematical perspective with the proofs of most of the results included: the only proofs which are omitted either (i) are left to the reader, (ii) are mentioned in other papers. Although it has a mathematical perspective, readers who are more interested in applied or computational aspects of the subject should find the explicit statements of the results helpful even if they do not concern themselves with the details of the proofs.

• Third, this thesis is meant to be a graduate requisite and not just a paper on the subject. This aspect of the thesis is reflected in the way the background materials are carefully reviewed as we use them. The ideas are introduced through numerical examples to learn the meaning of the theorems and master the techniques of the proofs and topic under consideration.

In this thesis we use difference equation in the kth order to introduce basic ideas and results of dynamical systems. In order to investigate this dynamical system we divided this thesis to four chapters:

Chapter 1 gives an introduction to dynamical systems, it gives some basic information to discrete system, linear system and difference equations. Chapters 2 shows in details the solutions of linear and nonlinear difference equations form the first up to $k^{th} order$.Chapter 3 shows in details the behavior of solutions of linear and nonlinear difference equations. Chapter 4 shows our problem in details; starting with the linearization and the equilibrium point, then conditions under which the equilibrium point will be local stable or global stable, and the others under which the solution will have period two solution, and finally we discuss the semicycles and invariant interval. The ideas are introduced through numerical examples to learn the meaning of the theorems and master the techniques of the proofs and topic under consideration.

As might be expected, the two cases $\mathbf{p} > \mathbf{q}$ and $\mathbf{p} < \mathbf{q}$ give rise to different dynamic behaviors.

We believe that the results about equation(1) are of paramount importance in their own right, the results presented also give the basic theory of the global behavior of solutions of nonlinear difference equations of order k. The techniques and results in this thesis are also extremely useful in analyzing the equations in the mathematical models of various biological systems and other applications. [7].